YACHT: Hull-Speed, Turbulence, Eddy-Viscosity and Energy-Consumption

Internal Friction in a Flow, Viscosity

Assume a (laminar) flow in z-direction driven by some constant force between two parallel plates at positions x=+b on one side and x=-b on the other side. At the plates the flow-velocity is zero, in the center at x=0 it is maximal. So the velocity-profile is approximately parabolic with a maximum $V(x=0) = Cb^2/\eta$. The constant C is proportional to the pressure-difference $\Delta p/\ell$ along the channel of length ℓ, acting as a force-density f~ρa with fluid-density ρ and acceleration a, and inversely proportional to a friction-constant $\eta=\rho\kappa$. In fact, η is called the dynamic viscosity, a materials parameter. Obviously the friction depends quadratically upon the plate distance b, so the damping rate upon the velocity or **kinematic viscosity κ** must have dimensons $[\kappa] = m^2/sec$. **Stokes'-Law:** When a sphere with radius r and density ρ_s falls into a liquid with lower density ρ, its sinking-speed is damped by a friction force $\mathbf{F= 6\pi \eta r V}$. Such a friction force acts also onto the hull of a yacht at speed V.

Turbulence, Reynolds-Number and Eddy-Viscosity.

A body with diameter L moving at velocity V within a flow generates rotating eddies in the flow. These eddies separate from the body and carry energy away, if the viscosity is small or the body L or the velocity V is large, which means "turbulence". It is measured by the dimensionless Reynolds-number $\mathbf{Re=L*V/\kappa}$. Turbulence exists when it exceeds a critical value of about Re>1000 (derivation complicated). Note that this occurs in air already at the normal speed of a pedestrian! In this turbulent case, obviously the normal viscosity **κ** should be replaced by the so-called **Eddy-viscosity** κ_E **= L*V**, as the energy-dissipation then mainly occurs via the generation of eddies carrying energy away from the moving body (which of course ultimately are damped by the normal viscosity).

Propulsion of a ship

A ship of size L driven by a propeller (and that driven by the engine) feels a friction-force in the water. In laminar flow this would be like in Stokes's-law F~κLV. But in practical situations the Reynolds number is always very large, so that the viscosity here must be replaced by the Eddy-viscosity, which in turn gives $\mathbf{F \sim V^2}$ for the friction force. So the energy dissipated by a ship moving over a distance d would be E=F*d, energy being force multiplied by distance. But the power-consumption P of the ship is measured in energy per time t, or P=E/t =F*d/t = F*V. So the power consumption then scales rather like $\mathbf{P \sim V^3}$. This is quite dramatic: reducing the initial speed of the ship by 50%, the power consumption goes down by a factor 8!

Hull-velocity

Of course the considerations above are subject to some conditions. Ordinary waves on a water surface are usually so-called gravitational waves. Any deformation on the water-surface will be counteracted by gravitation $g=9.81\ m/s^2$. Taking the length L of the ship as a multiplier, we arrive at a scaling relation V ~ *sqrt*(g*L) with *sqrt* meaning "square-root". In contrast to sound-waves or light-waves which propagate at constant speed, the speed of gravitational water waves depends on the wave-length. A slightly more detailed analysis gives the form $\mathbf{V_h}$**=*sqrt*(g*L/ 2π)** as the so-called hull-velocity of a ship which is not easily exceeded. This corresponds to a wave which has a maximum height at bow and stern, and a minimum in the middle. If one wants to accelerate the ship further one must heave it up onto to its own bow-wave. For a yacht of about 12 meters length the hull-velocity is about 4.32 m/s or 15.6 km/h. From experience one can estimate that the power-consumption to drive a yacht at its hull-speed is about 3kW/ton. So for a 10ton-yacht one would like to have about 30 kW of engine-power available. But note the energy-advantage to proceed at reduced speed!