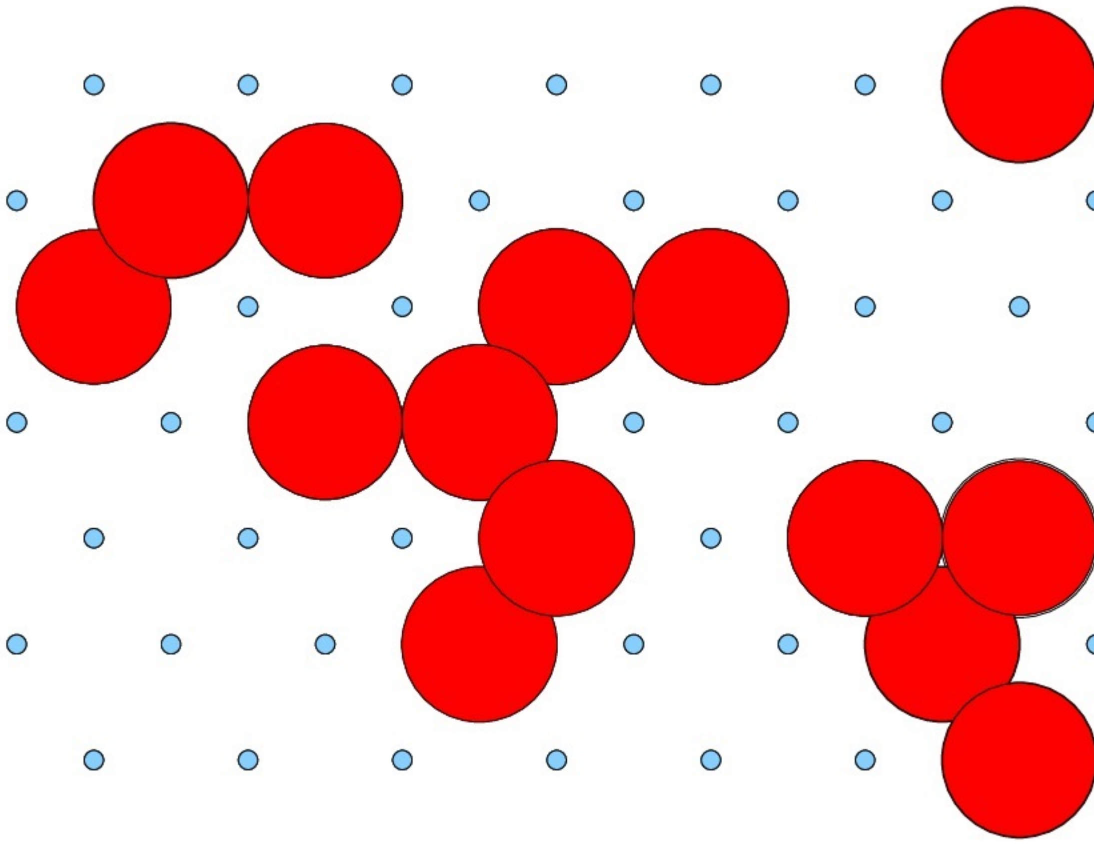
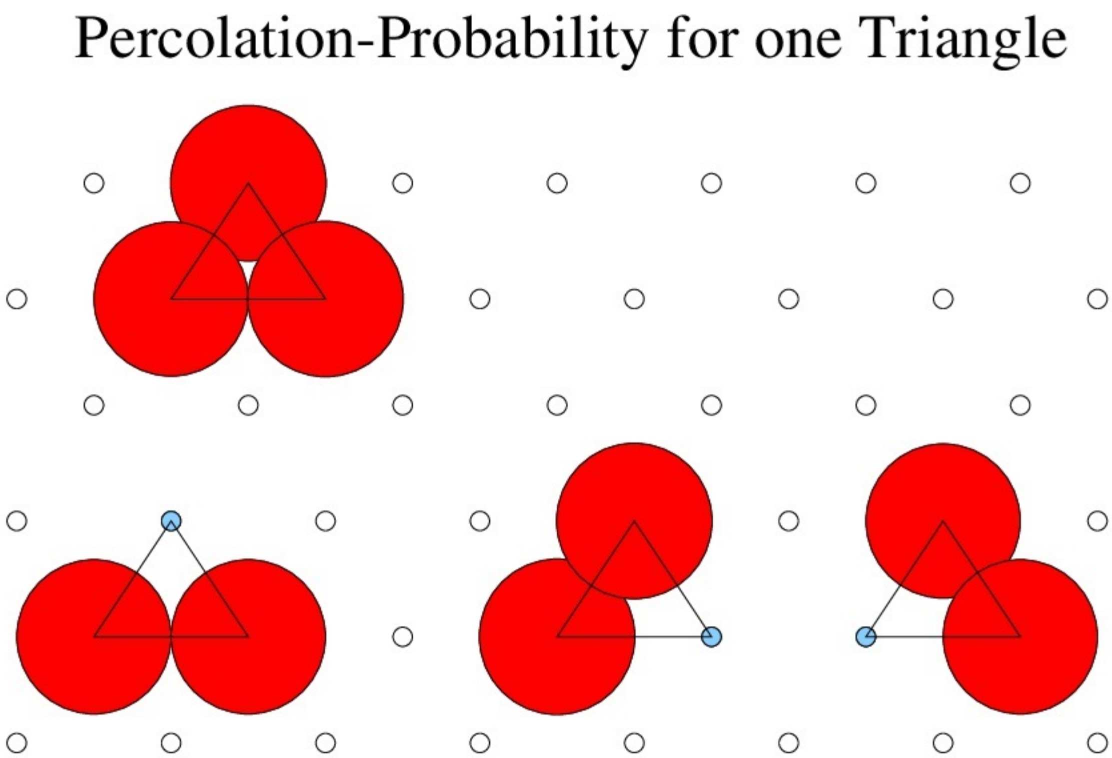
Percolation



Percolation on Triangular Lattice

Probability P that a blue grid-point is occupied by a red circle here is $P = 14/52 = 0.269$

Renormalization of Percolation



$$\tilde{P} = P^{3} + 3\,P^{2}\,(1 - P)$$

$\tilde{P}$ is the new effective probability after reducing the number of grid-points by three to one.

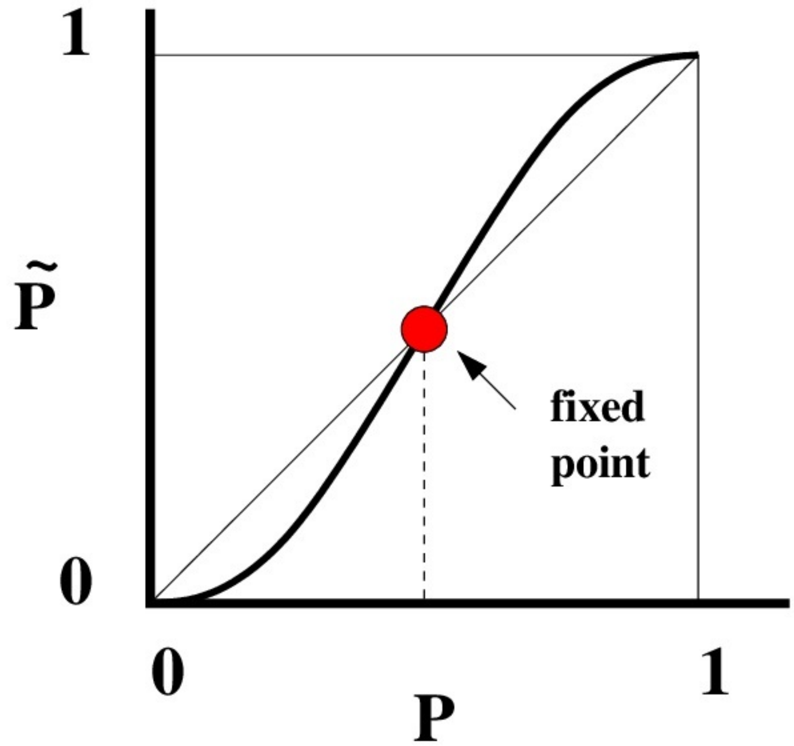
Renormalization Group

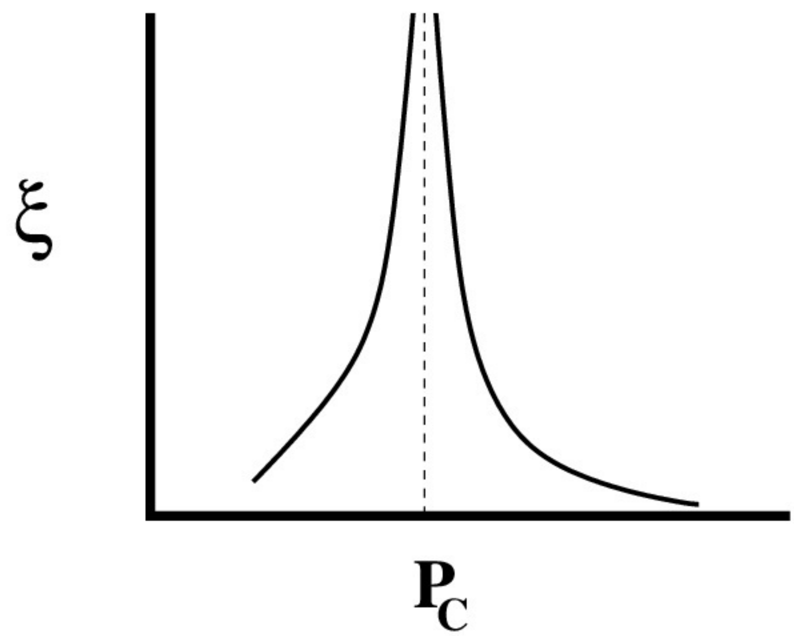
Example: Percolation
$P =$ probability, that one point is percolating
$\tilde{P} =$ probability, that a triangle is percolating
$\xi =$ diameter of typical percolation-cluster

Renormalize one triangle to one point:
$$\tilde{P} = P^3 + 3P^2(1 - P)$$
The compression of 3 points in the plane to 1 point is a compression of the area by a factor 3. This is a change in length scale by $b = \sqrt{3}$.
Renormalization-group transformation:
$$\tilde{P} = R(P); \qquad \tilde{\xi} = \xi \;/\; b$$
Fixed-point: $\tilde{P} = P = P_c = 1/2$ (and at 0,1).

The correlation-length ξ goes to infinity as $P \to P_c$.
Critical exponent ν (definition): $\xi \approx 1/|P - P_c|^\nu$
This gives $\xi/\tilde{\xi} = |(\tilde{P} - P_c)/(P - P_c)|^\nu$
and in numbers: $\sqrt{3} = (3/2)^\nu$, since near P_c we have the linearized renormalization transform:
$$\tilde{P} = R(P) \approx P_c + (3/2)(P - P_c).$$
The critical exponent ν therefore is:
$$\nu = log(\sqrt{3})/log(3/2) = 1.3547$$

Renormalization and Correlation Length





$\xi \sim 1/|P - P_c|^\nu$ with $\nu \approx 1.354$ is the correlation length ξ which goes to infinity at the percolation-threshold $P_c = 1/2$.